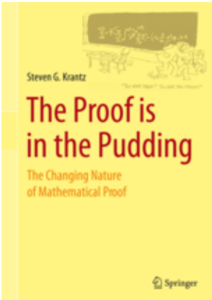
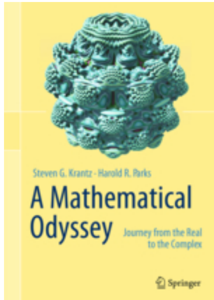
The Proof is in the Pudding, The Changing Nature of Mathematical Proof, 2011, Springer Verlag, ISBN 978-0-387-48744-1 (hbk), 264 pp. by *Steven Krantz*
A Mathematical Oddysey, Journey from the Real to the Complex, 2014, Springer Verlag, ISBN 978-1-4614-8938-2 (hbk), 398 pp. by *Steven Krantz & Harold R. Parks*





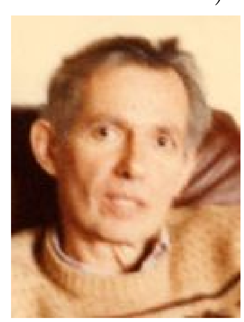
Steven Krantz

Of course the title of the first book is a wrong English idiom. You know it is incorrect, and yet you immediately know what is meant by it. Does this not remind of the way we read mathematical proofs? We know there are obvious steps omitted or the formulation is slightly wrong, but as long as we know what is meant we let is pass. So this brings us to the question: what is a mathematical proof, and when do we consider a proof to be correct.

The first book under review is an attempt to answer these questions. So the first chapter starts with some general ideas: the Platonic versus the Kantian view of mathematics, the role of conjectures, some elementary rules from (formal) logic, the role of (computer) experiments, etc. Then follows a sequence of chapters that follow more or less the historical development of mathematics. It starts with the Greek with their geometric approach to mathematics. The algebra, zero and infinity are heritages of the Arabs. Dirichlet is identified to lay the foundations of a rigorous proof (Although born in Düren, Germany, his full name is Peter Gustav Lejeune Dirichlet because his family is originally from Richelette, close to Liège, and hence he has some Belgian roots.) He was also the first to give a precise definition of a function. It is remarkable that Krantz makes Riemann a student of Dirichlet (p. 57), although all other sources identify Gauss as his supervisor. Gauss plays only a back plan role in this book, but so are many other big names.) The golden 19th century for mathematics is quickly skipped to move on with Hilbert and the 20th century. It is also the start of an American school of mathematics (Birkhoff, Wiener) where before it was France and Germany that formed the breeding ground for mathematical development. The American school was much more engineering (applied) oriented. (Krantz makes an interesting excursion to explain why America is currently the mathematical center of the world and compares the European and American tradition.) To return to the methods of proving: the Brouwer fixed point theorem has an existence proof that is not constructive although later in life he converted to constructivism. It is also a proof by contradiction, a proof technique later contested by Errett Bishop, the founder of constructive analysis. And of course the story of Nicolas Bourbaki and their approach to mathematics is fully told. An incentive to study the foundations of mathematics was given by several paradoxes (Bertrand, Banach-Tarsky, Monte Hall). The next step is the introduction of the computer in the history of the solution of the 4-colour problem, and the Turing machine leading to Gödel's incompleteness theorems. Krantz spends several chapters on the role of the computer in education, in proving new results, for generating graphics, and in typesetting mathematical papers. Famous problems get attention: classification of finite groups, the Bieberbach conjecture, the sphere packing problem, foliation theory, Perelman and the Poincaré conjecture, Riemann hypothesis, Goldbach conjecture, twin-prime conjecture,

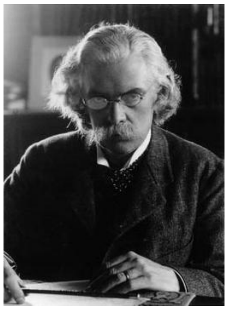
G. Lejeune Dirichlet

L.E.J. Brouwer



E. Bishop

Mandelbrot and fractals, the P/NP problem, and of course Wiles and his proof of Fermat's last theorem. This highlights again the problem of what is considered to be a proof. How about a proof by a computer? And what about proofs that are so long, deep and broad that no one referee will ever be able to confirm that the proof is completely correct?



G. Mittag-Leffler

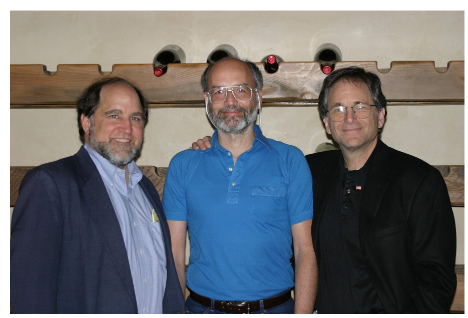
Alternatives for this book are for example [1, 2, 3, 4].

This is is one of the many books that Krantz has written (somewhere between 70 and 80 by now). It is addressing a reader that is generally mathematically interested. The most difficult and involving mathematical details themselves are avoided, but nevertheless, mathematics are definitely there and thus I believe that it will be mainly of interest to mathematicians. Some mathematical concepts are included as short inserts. On the other hand it is not only mathematics and there are remarkably few formulas. There are extensive historical parts with anecdotes. You can learn for example why there is no Nobel prize for mathematics. And it is not because a mathematician ran off with his wife as the wild story goes among mathematicians. The true reason, according to Kranz, is that Nobel was a solitary bachelor who was jealous of his mathematical compatriot Mittag-Leffler was a very exuberant and popular womanizer (he probably had an affair with Sonya Kovalevskaya). So probably Nobel did not create a prize for mathematics because the chance was high that Mittag-Leffler would get it.

The second book was co-authored by Steven Krantz and Harold Parks. Here the idea is to tell some of the success stories of mathematics since the 20th century. Fourteen cases are mentioned. Among these, we find the usual suspects, several of them also featured in the first book: the four colour problem (Apple & Haken 1976), mathematical finance with the Black-Scholes equation (1973), special relativity (Einstein 1905), RSA encryption (Rivest, Shamir and Adleman 1977), the P/NP problem, and Wiles proof of Fermat's last theorem (1995).

Black & Scholes

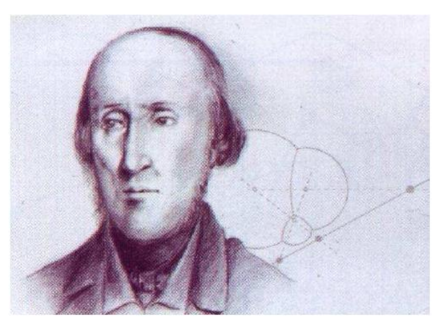
AKS



RSA (in 2003)

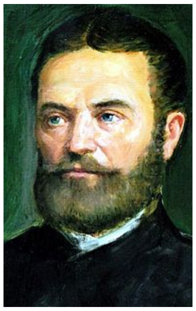
Also Perelman's adventures (1994) and his proof of Poincaré's conjecture is a well known story that has nearly obtained the status of a dramatic sitcom. Another topic is dynamical systems which is a pretext to review Mandelbrot sets (a 3D version is on the cover), the Lorenz attractor and other chaotic phenomena.

Wavelet theory is a marvelous example of how theoretical physics, signal processing, and harmonic analysis collaborated to boost this subject. As a result, it had an immediate impact on the applications which is rather unusual for mathematical theories. Minimal surfaces are approximated by soap films and bubbles, an observation by the Belgian physicist J. Plateau (1801-1883) and the problem was named after him. A proof was only given by J.E. Taylor in 1976. The Agrawal–Kayal–Saxena primality (AKS) test published in 2002 places this problem in the complexity class P.



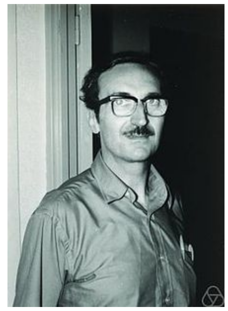
Joseph Plateau

The introduction of non-Euclidean geometry by J. Bolyai and N. Lobachevsky unchained vivid exploration of what had been a taboo for centuries. Riemannian geometry allowed Einstein to derive his relativity theory and nowadays Calabi-Yau manifolds are in the toolbox of theoretical physicists unraveling the tiniest building blocks of the physical reality. The chapter on Gödel's incompleteness theorem is rather extensive discussing formal axiomatic systems and computer generated proofs.



Janos Bolyai

Nikolai Lobachevski



Eugenio Calabi

Shing-Tung Yau

The chapters can be read independently. They differ a bit in style and extent. Some start with an extensive discussion of the early history. Others are rather short. Also the level of mathematical knowledge that is assumed differs. It is obviously intended to be accessible for a general readership, but I think that some chapters require from the reader a strong interest in the subject to keep on reading, if he or she is not mathematically trained. Every chapter ends with a section "A Look Back". Also that varies in length and content. Sometimes it is a kind of summary of the chapter, sometimes it gives additional or historical information. There are of course the recognizable characteristics of previous books by Krantz: the anecdotes, gossip, and inside stories about the main players and events. It should certainly give an idea to non-mathematicians, what keeps mathematicians busy all day, and why it is important that they do what they do.

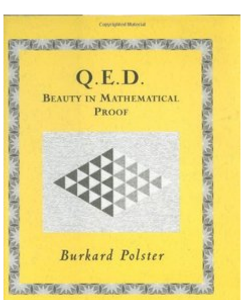
References

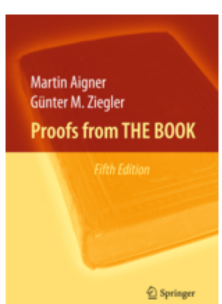
[1] Burkard Polster *Q.E.D.: Beauty in Mathematical Proof.* Wooden Books Ltd, 2006.
This is a short, nicely illustrated book with mainly visually oriented proofs. A new edition appeared in 2014.

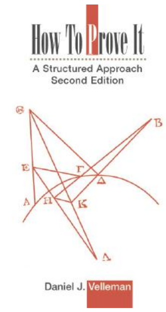
[2] Aigner, Martin & Ziegler, Günter *Proofs from THE BOOK.* Springer Verlag, 1998.
This well known book had updated editions in 2002, 2004, 2006, 2014.

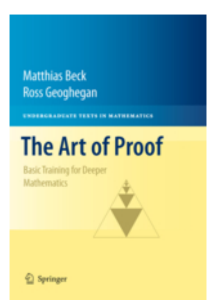
[3] Daniel J. Velleman *How to Prove It A Structured Approach.* Cambridge U. Press, 1994.
This explains how to break down a proof in elementary steps. (2nd ed. in 2006).

[4] Matthias Beck & Ross Geoghegan *The Art of Proof.* Springer Verlag, 2010.
A book to introduce undergraduates to methods for mathematical proofs.









Adhemar Bultheel